SUMMARY

A simulation-based method was developed to investigate the hazard of Wake Vortex Encounters (WVEs). This paper describes an important part of this method: the determination of worst-case WVE conditions, which is referred to as Worst-Case Search (WCS). The WCS results permit to reduce time and costs of WVE hazard related piloted simulator tests and allow the comparison of the hazard that vortices of different generator aircraft exert on a follower aircraft.

The WCS is based on a high fidelity, offline simulation of the follower aircraft that includes the interacting wake vortex, a hazard criterion that rates the severity of each WVE, and a pilot model. It can be formulated as an optimisation problem that is solved with the optimisation tool MOPS (Multi Objective Parameter Synthesis). MOPS varied the encounter geometry until the simulation yields maximum values of the WVE hazard criterion.

Worst-case encounter conditions for different parameters were investigated and sensitivity studies performed. The influence of the height above ground, the core radius, and the models for the wake vortex velocity profiles on the severity of a WVE was examined. To demonstrate the capability of the method, a comparison of the hazard of a WVE behind two different heavy transport aircraft for a 20t aircraft was made. The WCS method proved its applicability and delivered the worst-case encounter geometry.

NOMENCLATURE

Symbols

- $a$: Acceleration
- $b_v$: Spacing between wake vortices
- $C$: Criterion or constraint
- $C_{\text{high}}$: Upper criterion boundary
- $C_{\text{low}}$: Lower criterion boundary
- $\text{CRIT}$: Criterion quantity
- $H$: Height
- $H_{\text{GND}}$: Radio height above ground
- $K_C$: Gain of controlled element (aircraft)
- $K_P$: Pilot gain
- $p$: Roll rate
- $r_c$: Radius of wake vortex core
- $\text{SSPI}$: Side stick pitch input
- $\text{SSRO}$: Side stick roll input
- $T_{\text{lag}}$: Lag time constant
- $T_{\text{lead}}$: Lead time constant
- $T_R$: Roll time constant
- $t$: Time, separation time
- $V$: Speed
- $V_W$: Wind speed
- $\text{WV}_{\text{type}}$: Type of wake vortex velocity profile
- $x$: Coordinate system axis
- $Y_C$: Transfer function of controlled element (aircraft)
- $Y_P$: Transfer function of pilot
- $y$: Coordinate system axis
- $\gamma$: Lateral offset from the ILS
- $\Delta \Phi_{\text{WV}}$: Bank angle of wake vortex system
- $\Delta \gamma_{\text{WV}}$: Angle between inclination of ILS glide path and inclination of vortex axis (vertical encounter angle)
- $\Delta \Psi_{\text{WV}}$: Horizontal angle between ILS glide path and inclination of vortex axis (horizontal encounter angle)
- $\varepsilon^*$: Eddy dissipation rate
- $\gamma$: Flight path angle
- $\Phi$: Bank angle
- $\phi$: Circulation, vortex strength
- $\Theta$: Pitch angle
- $\Theta$: Equivalent time delay
- $\Omega$: Frequency
- $\omega$: Crossover frequency
- $\psi$: Azimuth angle

Subscripts

- crit: Critical
- EF: Encounter fix
- GND: Ground
- $G_{\text{GS}}_{\text{max}}$: Maximum glide slope (deviation)
- RWY: Runway
- $R CR_{\text{max}}$: Maximum roll control ratio
- WV: Wake vortex
- $\Phi_{\text{max}}$: Maximum bank angle
- 0: Initial

Abbreviations

- ATD: Advanced Technology Demonstrator
- GA: Go-Around
- GDEV: Glide slope Deviation in encounter plots
- GS: Glide Slope (ILS)
- ICAO: International Civil Aviation Organisation
- ILS: Instrument Landing System
- LDEV: Localiser Deviation in encounter plots
- LOC: Localiser (ILS)
- MOPS: Multi Objective Parameter Synthesis (program package)
- MLW: Maximum Landing Weight
- MTOW: Maximum Take-Off Weight
1. INTRODUCTION

1.1. Background

Every aircraft generates a pair of counter-rotating vortices as a consequence of lift generation. Trailing aircraft can inadvertently penetrate these vortices; especially in areas with dense air traffic, e.g. major airports or densely flown routes. Such Wake Vortex Encounters (WVEs) cause vortex-induced reactions of the trailing aircraft that can adversely affect flight safety. To avoid potentially hazardous WVEs, Air Traffic Control employs aircraft-to-aircraft separation standards. The current separation distances for landing approach depend on the maximum take-off weight (MTOW) of the generating and the trailing aircraft. According to the ICAO classification (FIG. 1), three aircraft categories are distinguished: Heavy (> 136 t), Medium (7-136 t) and Small/Light (< 7 t) [1]. Experience has shown that the existing separations are safe. However, they are considered to be rather conservative since essential effects are not taken into account. These include wake vortex characteristics, the influence of meteorological conditions and detailed flight mechanical characteristics of the follower aircraft including its control capabilities. As a consequence, current wake vortex separation distances limit airport and airway capacities greater than necessary.

In order to reduce separations to a safe minimum, the vortex influence on the aircraft and the corresponding hazard potential have to be investigated. The investigation of the wake vortex hazard in flight tests has been examined. Several types of instrumented aircraft probed vortex flow fields of known aircraft while encounter upsets were measured as a function of vortex strength and encounter geometry. Experience with such tests has shown that it is very difficult to control all of the important variables during such tests and to guarantee safety close to ground.

1.2. Methodology

As it has proved to be impractical to investigate WVEs in flight tests, a simulation-based methodology that relies on both piloted and offline simulations was developed. This methodology is called Vortex Encounter Severity Assessment (VESA) and its objective is a comparison of the WVE hazard that vortices of different generating aircraft exert on specific follower aircraft. Such a comparison allows assessing whether wake vortex separations are appropriate for new and larger aircraft.

The WVE hazard assessment methodology, as illustrated in FIG. 2, comprises five main steps:

1) Development of a WVE simulation
2) Piloted simulator tests to obtain subjective WVE hazard ratings
3) Development of WVE hazard criteria based on results of the simulator tests
4) Determination of worst-case encounter conditions and critical vortex strength on the basis of offline simulations using the hazard criteria
5) Encounter severity assessment based on Monte Carlo Simulations, i.e. comparison of hazard probabilities (e.g. of GA probabilities for landing approach) for different generator and follower aircraft combinations.

Worst-case encounter conditions are defined as a combination of parameter values for which a hazard level is exceeded for minimum vortex strength. The critical vortex strength is defined as minimum vortex strength for which a hazard level is exceeded for worst-case encounter conditions.

1.3. Objective

An important part of the WVE hazard assessment methodology is the Worst-Case Search (WCS) that
determines the worst-case encounter conditions and the critical vortex strength. The determination of both for an ILS approach is the objective of this study. The knowledge of the worst-case encounter conditions and the critical vortex strength has the following benefits:

- **Worst-case encounter conditions:** Within the described methodology piloted and offline simulations represent the crucial part. For safety relevant investigations they must cover the worst-case encounter conditions. To ensure that these critical encounter conditions are considered, extensive piloted simulator tests would be necessary. In order to avoid such time consuming and costly tests, worst-case encounter conditions have to be determined with an automated search method based on appropriate simulation models. The search results permit efficient piloted simulator tests by confining the test cases to the critical scenarios so that simulation time and costs can be significantly reduced.

- **Critical vortex strength:** The critical vortex strength can be used to compare the VWE related hazard that different generating aircraft exert on specific follower aircraft. This requires the assumption of certain atmospheric conditions and the application of a decay model (e.g. P2P [19] or from AVOSS [20]). The question that has to be answered is after which distance or time is the initial vortex strength decayed to the critical vortex strength. In the future, when wake vortex and decay models are more elaborated, the resulting distance or time can serve as a hazard measure: the longer the distance or time, the higher the hazard potential of a specific generator/follower aircraft combination.

Hereafter, the simulation and the WCS set-ups are explained, the worst-case encounter conditions are discussed for a VFW614-ATD behind a generic heavy transport aircraft and the potential of the WCS method to perform hazard comparisons is illustrated for the example of the VFW614-ATD behind two different generic heavy transport aircraft.

2. **SIMULATION**

The approach that is pursued for the determination of the worst-case encounter conditions is based on an offline simulation that is integrated into an optimisation routine. The offline simulation model, FIG. 2, consists of three submodels:

1) WVE simulation that comprises the flight mechanical model of the encountering aircraft, the wake vortex model and the aerodynamic interaction model
2) Pilot model
3) WVE hazard criteria.

2.1. **WVE Simulation**

For WVE simulations, a conventional six degrees of freedom flight simulation of the VFW614-ATD has been supplemented with the VWE software package [2]. The WVE software package, consisting of a wake vortex model and an aerodynamic interaction model, calculates additional vortex-induced aerodynamic forces and moments, which result from the WVE. For this calculation the strip method was used [3-5]. From the vortex-induced forces and moments translatory and angular wind velocity vectors are determined. They are equivalent to the vortex flow field, i.e. the non-uniform three-dimensional vortex flow field is transformed into an equivalent uniform flow field with constant and linear contributions. The equivalent flow field is represented by a translatory and rotatory wind velocity vector. These vectors generate (approximately) the same additional forces and moments, which the strip method has computed, when they are processed by the aerodynamic module of the base simulation [6].

As model for the wake velocity field, which is the main input of the strip method, either Winckelmans', Lamb-Oseen's, Burnham-Hallock's or Proctor's velocity profile can be used. The vortex strength is either selected arbitrarily in a realistic range or determined by WCS.

2.2. **Pilot Model**

In offline simulations the pilot's inputs for ILS tracking, aircraft stabilisation, and recovery during the WVE are simulated with a pilot model. Since most existing pilot models were developed for specific control tasks and for specific situations, the pilot model of the offline simulation consists of two sub modules, each for a specific control task:

- Pilot model for the WVE
- Pilot model for the ILS tracking.

2.2.1. **Pilot Model for ILS tracking**

The model for ILS tracking represents the behaviour of the pilot during the ILS approach. As pilot inputs for such a task have usually lower frequencies, it is also referred as the “low dynamics” part of the pilot model. FIG. 3 shows the inputs and outputs. Side stick pitch and roll commands and the throttle lever position are computed to correct localiser and glide slope deviations and to maintain the approach speed. More detailed information can be found in [2] and [7].

![FIG. 3: Block diagram of the pilot model for ILS tracking](image)

2.2.2. **Pilot Model for WVE**

The pilot model for WVE represents the pilot’s behaviour during a WVE [8]. For its development it was assumed that the primary effect of an encounter (with nearly parallel flight path and vortex axis) is a bank angle disturbance. Apart from the possible initiation of a GA, the pilot's
response to such a bank angle disturbance can be divided into two successive reactions: at first they re-establish wings level as quick as possible and, subsequently, they return to the ILS glide path.

The pilot model for WVE reflects the first reaction, which is assumed to be a compensatory control action using the bank angle as control variable. The model is based on the crossover law [9], i.e. it is quasi-linear and can be understood as an active single input and single output control element that operates on the bank angle. The crossover law states that the pilot adapts to the dynamics of the controlled element $Y_C$ (aircraft) in such a way that the dynamics of the open loop aircraft-pilot system $Y_pY_C$ can be approximated by the crossover frequency $\omega_c$ and the equivalent time delay of the aircraft-pilot system $\tau_e$ in the frequency range around 0 dB amplitude [10].

\[ Y_pY_C = \frac{\omega_c e^{-j\omega \tau_e}}{j\omega} \]

Assuming simple roll dynamics of the aircraft

\[ Y_C = \frac{K_C}{j\omega(1 + j\omega + \tau_e)} \]

the crossover law is fulfilled for a parametric pilot model equation

\[ Y_p = K_p \cdot \frac{T_{lead} \cdot j\omega + 1}{T_{lag} \cdot j\omega + 1} e^{\tau_e j\omega} \]

when $T_{lead} \approx T_R$, $K_p = \frac{K_C}{\omega_C}$ and $T_{lag} \approx 0$.

The time constants $T_{lead}$ and $T_{lag}$ and the time delay depend on the aircraft dynamics and have to be adapted to the simulated aircraft (for the VFW614-ATD: $T_{lead}=0.5s$, $T_{lag}=0.01s$, and $\tau_e=0.28s$). The gain of the pilot model $K_p$ has to be adapted to the roll dynamics of the controlled element and to the control task of the pilot (for the VFW614-ATD: $K_p = 0.04$). The block diagram of the WVE model is shown in FIG. 4.

![FIG. 4: Block diagram of the WVE pilot model](image)

The WVE pilot model calculates the side stick roll command SSRO from the bank angle input $\Phi$. Since the side stick roll command, i.e. the pilot’s reaction during the WVE, exhibits a higher frequency than the ILS tracking actions, the pilot model for WVE is also denoted as "high dynamics model".

2.2.3. Pilot Model Management

When the two pilot models are used for WCS simulations it is important to avoid any interaction between both models. The following logic prevents the models from being active at the same time:

- Low dynamics ILS model ON and high dynamics crossover model OFF: if $p < 5°/s$
- Low dynamics ILS model OFF and high dynamics crossover model ON: if $p > 5°/s$
- If the high dynamics model is switched ON, $p$ has to be smaller than 5°/s for at least three seconds in order for switching the low dynamics model ON and the high dynamics model OFF.

The transition phase of the switching process between the high and low dynamics model takes 200 ms. Tests have shown that the transition between the pilot models did not cause any transients.

2.3. Hazard Criteria

Wake vortex hazard criteria – or simply hazard criteria – determine whether an aircraft on a given flight path will be sufficiently disturbed by a vortex encounter so that it could be dangerous to continue the mission, i.e. hazard criteria quantify the WVE severity and relate the hazard that is perceived by the pilot to objective measurable data, e.g. maximum bank angle or glide slope deviation.

Within the S-WAKE project, Airbus developed four hazard criteria for the VFW614-ATD and landing approach flight phase [12, 13]: bank angle criterion, roll control ratio (RCR) criterion, glide slope criterion and the combined RCR and glide slope criterion (RG criterion). Among these criteria, the RG criterion achieved the best predictions – but as the RCR is inherently related to the distance of the encountering aircraft to the vortex axis and does not consider the time that the aircraft spends in the region with large induced rolling moments, it was not suited for optimisation and WCS [14]. Therefore, the bank angle criterion was used for all WCS (active criterion). All other criteria were calculated and monitored but had no influence on the optimisation and the results (passive criteria). In the following only the bank angle criterion is described. Descriptions of the other criteria can be found in Ref. [12, 13].

FIG. 5 shows bank angle criterion values in a plot of the height above ground for the maximum bank angle versus the maximum bank angle. Each symbol represents an individual WVE. The boundary for the VFW614-ATD, indicated as solid line, divides the criterion plot into two regions: a GA and a NOGA region (piloted simulator tests have shown that the GA decision serves as an indicator for a certain hazard level of the WVE during landing approach [12]).

For comparison, boundaries of the Boeing 707 (dashed line) and the Learjet (dash-dot line), which result from NASA investigations [15, 16], are also shown in FIG. 5. These boundaries were derived in a similar but slightly different way: pilots rated an encounter as hazardous or non-hazardous and were not asked to perform a GA. However, all boundaries illustrate that pilots tolerate larger vortex-induced maximum bank angle values with increasing height above ground.

The equation of the criterion boundary for the VFW614-ATD
(4) $|\Phi_{\text{max}}| = 0.178 \cdot H_{\Phi_{\text{max}}} + 3.25$

can be used to normalise the bank angle criterion:

(5) $\text{CRIT}_{\Phi} = \frac{\Phi}{0.178 \cdot H + 3.25}$

This expression clearly differentiates between GA and NOGA:

GA prediction: $\text{CRIT}_{\Phi} > 1$

NOGA prediction: $\text{CRIT}_{\Phi} < 1$

FIG. 5: Bank angle criterion (test cases from VFW 614 ATD simulator tests)

The criterion evaluation has to ensure that the critical metric values are used. How these critical values are determined is described in FIG. 6, which shows a potential evolution of the bank angle during flight.

FIG. 6: Evolution of $\Phi$ as a function of time

The maximum bank angle $\Phi_{\text{max}}$ is reached at point 1 without violating the criterion boundary. A few seconds later, at 100 m, the boundary is crossed (point 2) and the maximum violation of the boundary is reached at point 3. Consequently, point 3 represents the critical combination of the height above ground and the bank angle. During WCS the maximum violation of the criterion boundary (in case of a GA) or the point that exhibits the closest distance to the criterion boundary (in case of NOGA), respectively, is relevant. The corresponding values are determined as maximum of equation 5 during each simulation.

3. APPLICATION OF MOPS TO WORST-CASE SEARCH

WCS is formulated as an inverse optimisation problem. The objectives of this optimisation problem are expressed as mathematical criteria and constraints, which represent quantitative measures for the assessment of the WVE simulation. With increasing complexity, i.e. an increasing number of conflicting criteria and constraints, which have to be fulfilled simultaneously, the solution of such a multi-objective optimisation problem becomes more and more difficult. Therefore, the application of a suitable software package is necessary. For the current study the software tool MOPS (Multi-Objective Parameter Synthesis, developed by DLR [17, 18]) was chosen.

3.1. MOPS

MOPS supports the engineer in properly setting and solving multi-objective optimisation tasks. The program package was initially developed for control system design but has recently proved its applicability in other domains as well. MOPS offers criteria libraries, a generic multi-model structure for multi-disciplinary problems, a generic multi-case structure, optimisation subroutines and visualisation tools for monitoring the optimisation process. Several additional features for dealing with a large amount of parameters and criteria, distributed computation for time consuming computations, and the use of external simulation and analysis servers are also available. The user communicates via an application program interface and a graphical user interface. MOPS is implemented in MATLAB. It is able to address all criteria and constraints simultaneously, while compromising them individually according to given demand [18].

3.2. Criterion Formulation

In order to solve the WCS problem with MOPS, it was formulated as follows: determine the parameter (tuner) values, for which minimal vortex strength is just sufficient to initiate a GA. In other words, the vortex strength is a criterion, which has to be minimised while the hazard criterion, which represents the GA decision, has to be interpreted as constraint. This constraint provides the information, for which vortex strength and for which set of tuners a GA is just initiated.

Within the MOPS environment, the vortex strength has to be defined as criterion of the type "minimum". The hazard criterion has to be defined as constraint of the type "inequality" [17]. The normalised bank angle criterion can be formulated in the following way:

NOGA: if $C_{\text{low}} < \text{CRIT}_{\Phi} < C_{\text{high}}$

GA: if $\text{CRIT}_{\Phi} < C_{\text{low}}$ or if $\text{CRIT}_{\Phi} > C_{\text{high}}$

$C_{\text{low}}$ and $C_{\text{high}}$ define the lower and upper boundary of the constraint $\text{CRIT}_{\Phi}$. 
To compare criteria for a multi-objective optimisation problem, a proper normalisation of the criteria is necessary via appropriate transformations (e.g. scaling and shifting). MOPS provides a convenient framework to automatically normalise criteria on the basis of specified good/bad limiting values. The so-called bad/good transformation maps $\text{CRIT}_b$ to a normalised function $C_1$, for which $+1.0$ separates 'good' (here: NOGA) and 'bad' values (here: GA), FIG. 7. This leads to the simple expression:

NOGA: \[ C_1 = \begin{cases} 1.0 & \text{if } C_1 < 1.0 \\ \text{bad GA} & \text{else} \end{cases} \]

FIG. 7: Bad/good transformation

As here a worst case has to be found, this equation has to be inverted:

NOGA: \[ C_1 = \begin{cases} 2.0 - C_1 & \text{if } C_1 > 1.0 \\ \text{bad GA} & \text{else} \end{cases} \]

FIG. 8 shows the second transformation. The optimiser will try to reach the minimal allowed values of $C_2$, which are located (and indicated) on the boundaries.

FIG. 8: Second transformation

3.3. Worst-Case Search Procedure

MOPS provides methods and a specialised environment (GUI, interfaces, process communication) for solving multi-objective optimisation problems. Its open architecture allows coupling of existing simulation software written in MATLAB/SIMULINK or in any other computer language. This capability of MOPS facilitated the implementation of the high fidelity, offline model, which is written in Fortran.

FIG. 9 gives an overview on how the model was integrated into the MOPS environment for the WCS.

The WCS procedure is defined in a specific MOPS/MATLAB script called runscript. It starts with defining initial tuner values, which are used to automatically generate input files for the simulation. Then, the runscript starts the simulation model, which is the offline model that was previously described. It uses the input files to initialise the simulation, to define the vortex geometry, and to control the simulation. The simulated scenario is as follows:

- The aircraft, the VFW614-ATD, is trimmed and stabilised on the ILS glide path outside the influence of the vortex.
- The aircraft is descending on the ILS glide path and approaching the vortex, which is positioned on or close to the glide path.
- Vortex-induced forces and moments disturb the aircraft; the pilot model tries to maintain the glide path and to recover from aircraft banking, whilst penetrating the vortex.
- The simulation stops if the aircraft has left the vortex influence zone, which is defined by a 64 m tube around the vortex axis.

During the simulation time histories are recorded. They are used to compute criteria values, which can be monitored during the WCS.

The applied optimisation method uses the outcome of the criteria assessment to compute new tuner values and the optimisation cycle is repeated. The WCS is finished when either the worst-case has been found or the maximum number of iterations is reached.

MOPS offers different optimisation algorithms: sequential quadratic programming, quasi-Newton method, pattern search, simplex method, generic algorithms, and simulated annealing. The WCS was done with pattern search, as it is a derivative free method that uses only function evaluations and as it is often numerically more robust than gradient-based methods.

4. WORST-CASE ENCOUNTER CONDITIONS

4.1. Test Cases

TAB. 1 gives an overview of the WCS tests that were performed. The first seven test cases investigate the impact of different tuners separately (single-tuner computations). Tuners are parameters, which influence the hazard of a WVE during landing approach and for which the worst-case values are determined. In test cases 8 to 9, two or more tuners are varied simultaneously, taking the results of the single-tuner computations into account. Column 3 shows the nominal values of the respective tuners and column 4 lists the corresponding
parameter range that is considered for WCS.

In FIG. 10 all tuners are visualised. In addition to the tuners, the vortex strength \( \Gamma_{\text{VW}} \) and the lateral offset between ILS glide path and vortex axis at the encounter fix \( y_{\text{EF}} \) are shown. It has to be noted that the vortex strength, which has a direct impact on the severity of a WVE, is not used as a tuner but as a criterion (as explained above).

The auxiliary tuner was necessary since WVE simulations are constant and not statistically varied as would be the case in reality.

4.2. Worst-Case Search

Before the optimisation method (pattern search) had been used for WCS, a sensitivity study was performed by systematically varying two or more tuners: equally spaced grid points for all active tuners were defined and MOPS computed the criteria values for each tuner value combination (gridding method). Subsequently, MOPS compared the results and indicated the worst of all gridding cases. This investigation provided physical insight by relating tuner values and encounter severity. Furthermore, it delivered tuner values, which served as start values for the following optimisation.

As an example, the WCS for test 8 is shown in the following. The settings for the gridding run are as follows:

- **Follower aircraft**: VFW614-ATD
- **Flight phase**: landing approach
- **Vortex strength**: kept constant at 134 m\(^2\)/s
- **Height of encounter fix**: \( H_{\text{EF}} = 220 \) ft
- **Tuners**: \( \Delta \Psi_{\text{VW}} \) varied in the range \( [0^\circ \ldots 20^\circ] \) using 12 grid points; \( \Delta \gamma_{\text{VW}} \) varied in the range \( [-5^\circ \ldots 1^\circ] \) using 5 grid points; \( y_{\text{EF}} \) varied in the range \( [15 \ldots 60m] \) using 14 grid points

In FIG. 11 and 12, the diagrams of the parallel coordinates and the combined time history and criteria plot, the results for all 840 tuner value combinations of the gridding run are plotted. FIG. 11 shows the normalised vortex strength and the four criteria (in optimisation context called *constraints*) in parallel coordinates. The different colours of the vertical axes symbolise the type and status of the criteria/constraints with green for an active criterion, blue for a passive constraint, and red for an active constraint. The lowest value of the bank angle criterion indicates the worst case of the 840 tuner value combinations (bold red line). As the value of the bank angle criterion is greater than 1.0, it is an encounter where the pilot would not perform a GA.

To allow arbitrary flight paths through the vortex, \( y_{\text{EF}} \) is used as an auxiliary tuner, i.e. \( y_{\text{EF}} \) was always active in addition to the active tuners from TAB. 1. During the optimisation \( y_{\text{EF}} \) forces the aircraft to penetrate the vortex in the most adverse way so that the wake vortex penetration can be considered as worst-case penetration. The auxiliary tuner was necessary since WVE simulations with the pilot model lead to deterministic aircraft responses, which are repeatable as the pilot parameters are constant and not statistically varied as would be the case in reality.

**TAB. 1: Overview of WCS test cases**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Active Tuner</th>
<th>Nominal Value</th>
<th>Range</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Delta \Psi_{\text{VW}} )</td>
<td>0°</td>
<td>0…20°</td>
<td>All other passive tuners have nominal values</td>
</tr>
<tr>
<td>2</td>
<td>( \Delta \gamma_{\text{VW}} )</td>
<td>-3°</td>
<td>-5…-1°</td>
<td>Worst-case value from test 1, all other passive tuners have nominal values</td>
</tr>
<tr>
<td>3</td>
<td>( H_{\text{EF}} )</td>
<td>220 ft</td>
<td>220…500 ft</td>
<td>Worst-case values from test 1 and 2, all other passive tuners have nominal values</td>
</tr>
<tr>
<td>4</td>
<td>( b_v )</td>
<td>10 m</td>
<td>16…80 m</td>
<td>Worst-case values from test 1 to 3, all other passive tuners have nominal values except ( r_c = 5% \times 4/\pi \times b_v )</td>
</tr>
<tr>
<td>5</td>
<td>( \Delta \Phi_{\text{VW}} )</td>
<td>0°</td>
<td>-25…25°</td>
<td>Worst-case values from test 1 to 3, all other passive tuners have nominal values</td>
</tr>
<tr>
<td>6</td>
<td>( WV_{\text{freq}} )</td>
<td>4</td>
<td>1, 2, 3, 4</td>
<td>Worst-case values from test 1 to 3, all other passive tuners have nominal values</td>
</tr>
<tr>
<td>7</td>
<td>( r_c )</td>
<td>2.9 m</td>
<td>2…5 m</td>
<td>WV_{\text{freq}} = 2, worst-case values from test 1 to 3, all other passive tuners have nominal values</td>
</tr>
<tr>
<td>8</td>
<td>( \Delta \Psi_{\text{VW}} )</td>
<td>0…20°</td>
<td>0…-1°</td>
<td>( H_{\text{EF}} = 220 ) ft, all other passive tuners have nominal values</td>
</tr>
<tr>
<td>9</td>
<td>( \Delta \gamma_{\text{VW}} )</td>
<td>0…-1°</td>
<td>0…-1°</td>
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</tr>
</tbody>
</table>

**FIG. 10: Visualisation of the tuners**

Before the optimisation method (pattern search) had been used for WCS, a sensitivity study was performed by systematically varying two or more tuners: equally spaced grid points for all active tuners were defined and MOPS computed the criteria values for each tuner value combination (gridding method). Subsequently, MOPS compared the results and indicated the worst of all gridding cases. This investigation provided physical insight by relating tuner values and encounter severity. Furthermore, it delivered tuner values, which served as start values for the following optimisation.

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To allow arbitrary flight paths through the vortex, \( y_{\text{EF}} \) is used as an auxiliary tuner, i.e. \( y_{\text{EF}} \) was always active in addition to the active tuners from TAB. 1. During the optimisation \( y_{\text{EF}} \) forces the aircraft to penetrate the vortex in the most adverse way so that the wake vortex penetration can be considered as worst-case penetration. The auxiliary tuner was necessary since WVE simulations with the pilot model lead to deterministic aircraft responses, which are repeatable as the pilot parameters are constant and not statistically varied as would be the case in reality.

**FIG. 11: Parallel coordinates for results of gridding run**

For the same sensitivity study FIG. 12, the combined time history and criteria plot, shows the evolution of the most important time histories (bank angle, glide slope deviation, radio altitude, and RCR) and the four criteria plots (bank angle, glide slope deviation, RCR, and RG criterion). In the
time histories, it is indicated when each criterion reaches its maximum by the corresponding symbols from the criterion plot. For each iteration step the combined time history and criteria plot is complemented with the actual curves and points so that all iteration steps are displayed when the optimisation is finished. All curves and points belonging to the same iteration automatically get the same colour (this is also true for the diagram of the parallel coordinates).

The worst-case tuner values found by gridding were used as start values of the optimisation with the pattern search method. The maximum bank angle criterion was used as active constraint. The optimisation led to the following results:

$$\Delta \Psi_{WV} = 14.0^\circ \quad \Delta \gamma_{WV} = -1.0^\circ \quad y_{EF} = 46.6\text{m}$$

The critical vortex strength for the initialisation of a GA is 135.9 m\(^2\)/s. As expected this value is slightly larger than the value of 134 m\(^2\)/s, which was used during the gridding and which did not cause a GA.

The results of the pattern search indicate that the gridding method predicted the worst case very well. FIG. 13 and 14 show that for the highlighted worst case the GA boundary is just reached for minimal vortex strength. The optimisation has fine-tuned the gridding results and has delivered the critical vortex strength.

4.3. Results

Analogous to the example of section 4.2, the results for the other ten test cases were computed. **TAB. 2** summarises these results. The investigation improved the understanding of encounter physics. Results of test 1 confirm simulator observations, results of test 2 and 3 are trivial, and results of test 4 to 7 indicate a close relationship between the geometric properties of the following aircraft and the worst-case values. For test 4 the worst-case vortex spacing of 20.4 m is similar to the wingspan of the VFW614-ATD of 21.5 m, so that a relation of both quantities can be assumed.

**FIG. 12:** Combined time history and criteria plot for results of gridding calculation

**FIG. 13:** Parallel coordinates for results of pattern search

**FIG. 14:** Combined time history and criteria plot for results of pattern search
The decrease of the critical vortex strength from test 3 to test 5 is attributed to a specific and hypothetical combination of parameter values – especially of $\Delta \Phi_{WV}$ and $b_v$ – where the follower aircraft penetrates not only one vortex but also the influential sphere of the second vortex (double encounter). The effects of the second vortex and pilot inputs have an adverse effect on the wake vortex encounter. As the pilot model was not validated for double encounters, this result is preliminary and has to be treated with care.

The comparison of the single- and multi-tuner WCS (test 8 and 9) shows that the simultaneous optimisation of different tuners leads to similar results. Only a slight reduction of the critical vortex strength due to a mutual dependency of the worst-case values becomes apparent when test 2/3 and 5 are compared with test 8 and 9, respectively. If the vortex model by Winckelmans and nominal vortex spacing are taken into account, the smallest vortex strength was determined as 113.5 m$^2$/s for test 9.

5. AIRCRAFT COMPARISON

This section shows an exemplary comparison of the wake vortex hazard that the vortices of two different heavy transport aircraft exert on a small transport aircraft. However, the results of this example are only preliminary. The intention of this chapter is to illustrate the potential that the WCS method has when vortex wake and decay models have improved in the future.

5.1. Test Case

The VFW614-ATD offline simulation was again used to simulate the follower aircraft during the landing approach flight phase. The height of the encounter fix was constant at 220 ft. As generator aircraft, two generic heavy transport aircraft were used. Their wake vortex velocity profiles were described by Winckelmans’ model. The characteristic vortex parameters of the generator aircraft are given in TAB. 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Generator A</th>
<th>Generator B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_v$ [m]</td>
<td>45.6</td>
<td>50.58</td>
</tr>
<tr>
<td>$r_c$ [m]</td>
<td>2.9</td>
<td>3.22</td>
</tr>
<tr>
<td>$\Gamma_{0MLW}$ [m$^2$/s]</td>
<td>354.2</td>
<td>449.9</td>
</tr>
</tbody>
</table>

TAB. 3: Vortex data of the generator aircraft

Test case no. 9 from TAB. 1 was used to determine the worst-case values of $\Delta \Phi_{WV}$, $\Delta \gamma_{WV}$ and $\Delta \Psi_{WV}$ and the critical vortex strengths for both generator aircraft A and B.

5.2. Results

The WCS results indicate similar worst-case values for both generator aircraft (TAB. 4). However, this is not always the case. Variations of $\pm 2^\circ$ in $\Delta \Phi_{WV}$ and much larger variations for $\Delta \Psi_{WV}$ have been observed for other aircraft combinations.

<table>
<thead>
<tr>
<th>Generator Aircraft</th>
<th>Worst-Case Values</th>
<th>Aux. Tuner $y_{EF}$ [m]</th>
<th>$\Gamma_{crit}$ [m$^2$/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$16.5 \pm 1$</td>
<td>7.7</td>
<td>54.5</td>
</tr>
<tr>
<td>B</td>
<td>$16.4 \pm 1$</td>
<td>7.7</td>
<td>52.4</td>
</tr>
</tbody>
</table>

TAB. 4: WCS results for aircraft A an B; $H_{EF} = 220$ ft

As expected, the critical vortex strengths of both generator aircraft are different. However, to assume the conclusion that the aircraft with the smaller critical vortex strength exerts the higher hazard potential is wrong given that the critical vortex strength has to be always related to the initial vortex strength. The crucial question that has to be answered for both aircraft combinations (VFW614-ATD behind heavy generator aircraft A and B) is: How long does it take and what separation distance is required until the vortex has decayed below the critical vortex strength $\Gamma_{crit}$ i.e. $\Gamma/\Gamma_{crit} = 1$? The resulting separation distance or time is called critical distance or time and represents a measure for the hazard that a generator aircraft exerts on the follower aircraft.

In order to determine the critical distance or time, the decay has to be considered. For this exemplary hazard comparison Sarpkaya’s decay model was used [20]. The application of this model required assumptions on meteorological conditions such as the turbulence level. FIG. 15 shows the decay functions for generator aircraft A and B and a medium to strong turbulence level that is defined by the normalised eddy dissipation rate $\varepsilon^*$ of 0.25. As initial vortex strength, the circulation for maximum
landing weight, $\Gamma_{\text{GW}}$, was assumed (worst case), TAB. 
3. However, according to Sarpkaya’s decay model the displayed decay curves are only valid until $T^* = 2\pi b^2/\Gamma_0$ (indicated by $\bullet$ and $\ast$). Thereafter, the curves are extrapolated. The extrapolation is conservative since in reality a faster decay can be expected.

FIG. 15 indicates different critical times for both generator aircraft ($t_A$ and $t_B$ for aircraft A and B, respectively) with aircraft B exhibiting a higher hazard potential than generator aircraft A since $t_A < t_B$. Furthermore, FIG. 15 emphasizes the improvement made to determine the critical vortex strength for each generator/follower aircraft combination separately. Idetical critical vortex strengths can lead to results that are more conservative than necessary. For example, the assumption of the critical vortex strength of aircraft A as safety limit for both generator aircraft would lead to the critical time $t_A'$ that is significantly larger than $t_B$ and, therefore, too conservative. Hence, it is recommended to perform any wake vortex hazard comparison on the basis of “identical encounter separation times or distances” rather than on “identical (critical) vortex strength separation times or distances”.

![Decay curves for generator aircraft A and B based on Sarpkaya’s decay model; $H_{\text{EF}} = 220$ ft](image)

**FIG. 15:** Decay curves for generator aircraft A and B based on Sarpkaya’s decay model; $H_{\text{EF}} = 220$ ft

6. CONCLUSIONS

From the described WCS studies the following conclusions can be drawn:

- A WCS method was developed and successfully applied to WVEs of the VFW614-ATD. The results are reasonable and confirm the observations during the Airbus simulator tests [12]. They give insight into the complex relationships of the WVE conditions. The knowledge of the worst-case encounter conditions permits a reduction of future WVE simulator tests to critical scenarios and to perform the test program more efficiently.

- Seven parameters were varied. Their ranges were defined such that realistic conditions are covered. WCS results are valid within these ranges.

- It became obvious that the severity of a WVE increased with decreasing encounter height, down to a 220ft, where the simulation and the criterion computation are not valid anymore.

- The core radius has a strong impact on the results. The smaller it is, the higher the hazard potential of a WVE. This relates to the characteristics of the vortex model: a reduction of the core radius increased tangential velocities close to the vortex axis without decreasing the velocities further outside.

- The models for the wake vortex velocity profile had a significant impact on the severity of the WVE. Following models are listed in order of decreasing severity of the WVE: Lamb-Oseen, Proctor (+20% $\Gamma_{\text{crit}}$), Winckelmanns (+20% $\Gamma_{\text{crit}}$), Burnham-Hallock (+37% $\Gamma_{\text{crit}}$). [* with respect to Lamb-Oseen]

- The comparison of the single- and multi-tuner WCS has show that the simultaneous optimisation of different parameters leads only to a slight reduction of the critical vortex strength.

- The simultaneous variation of $\Delta \Psi_{\text{WV}}$, $\Delta \Phi_{\text{WV}}$, and $b_V$ revealed a mutual dependency of the worst-case values of these tuners. For other tuners such dependency was not found.

- The worst-case encounter geometry was found for the following parameter values: $\Delta \Psi_{\text{WV}} = 16.5^\circ$, $\Delta \Phi_{\text{WV}} = -1^\circ$ and $\Delta \Phi_{\text{WV}} = 7.7^\circ$ and a critical vortex strength of $\Gamma_{\text{crit}} = 113.5$ m²/s (test 11).

- With the help of the critical vortex strength, which is a further result of the WCS, and a decay model the hazard that vortices of different generator aircraft exert on a follower aircraft can be determined (taking into account worst-case encounter conditions). Due to this capability the WCS method has the potential (when wake vortex and decay models have improved in the future) to be used to determine safe aircraft separation distances or times.

- The comparison of two aircraft regarding the WVE hazard potential should be based on identical encounter severity that they exert on trailing aircraft. A comparison based on initial vortex strength does not correctly predict the actual hazard.

ACKNOWLEDGEMENT

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REFERENCES


